and since x^2 is a square it only remains to make $2x$ a square, which it is when $x=2$. But this value of x makes the numbers the same.

Our next value of x is 8 and $x^2=64$, and $2x=16$, which numbers answer the conditions. The next value of x is 18 and the numbers are 324 and 36, and so on *ad infinitum.*

Also solved by *W. H. DRAUGHON, ARTEMAS MARTIN, F. P. MATZ, J. F. W. SCHEFFER, G. B. M. ZERR,* and *J. K. ELLWOOD*.

PROBLEMS.

13. **Proposed by ARTEMAS MARTIN, LL. D., U. S. Coast and Geodetic Survey Office, Washington, D. C.**

It is is required to find four numbers the sum of whose fourth powers is a square number.

14. **Proposed by SYLVESTER ROBINS, North Branch Depot, New Jersey.**

Find initial terms in each of three infinite series of prime, integral, rational, scalene triangles, where 9 shall be the base, and the other two sides of every term shall have a constant difference.

15. **Problems, or Propositions by M. A. GRUBER, M. A , War Department, Washington. D. C.**

(*a*) The *difference* of two *odd* squares is always divisible by 8. Corollary: Every odd square is of the form $8a+1$.

(*b*) The *sum* of two *odd* squares is two times an *odd* number.

Solutions to these problems should be received on or before November 1st.

AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Kidder, Mo. All Contributions to this department should be sent to him.

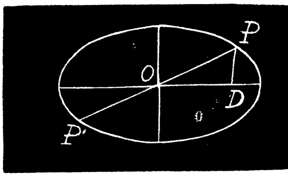
SOLUTIONS TO PROBLEMS.

6. **Proposed by J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.**

Find the average length of all the diameters that can be drawn in a given ellipse.

II. **Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland, and the PROPOSER.**

By using the *complement* of the eccentric angle we deduce $OD=a\sin\phi$



$PD=b\cos\phi$, $PP'=2a\sqrt{(1-e^2\cos^2\phi)}$=any diameter, and $ds=\sqrt{(1-e^2\sin^2\phi)}d\phi$. Hence the required average length becomes

$$D=2a^2\int_0^{\frac{1}{2}\pi}\sqrt{[(1-e^2\cos^2\phi)(1-e^2\sin^2\phi)]}d\phi$$

$$\div a\int_0^{\frac{1}{2}\pi}\sqrt{(1-e^2\sin^2\phi)}d\phi$$

$$=\frac{2a}{E(e,\frac{1}{2}\pi)}\int_0^{\frac{1}{2}\pi}\sqrt{[1-e^2(\sin^2\phi+\cos^2\phi)+e^4\sin^2\phi\cos^2\phi]}d\phi$$

$$=\frac{a(2-e^2)}{E(e,\frac{1}{2}\pi)}\int_0^{\frac{1}{2}\pi}\sqrt{\left[1-\left(\frac{e^2}{2-e^2}\right)^2\cos^2 2\phi\right]}d\phi$$

$$=\frac{-\frac{1}{2}a(2-e^2)}{E(e,\frac{1}{2}\pi)}\int_{+\frac{1}{2}\pi}^{-\frac{1}{2}\pi}\sqrt{\left[1-\left(\frac{e^2}{2-e^2}\right)^2\sin^2(\tfrac{1}{2}\pi-2\phi)\right]}d(\tfrac{1}{2}\pi-2\phi)$$

$$=\frac{\frac{1}{2}a(2-e^2)}{E(e,\frac{1}{2}\pi)}\int_{-\frac{1}{2}\pi}^{+\frac{1}{2}\pi}\sqrt{\left[1-\left(\frac{e^2}{2-e^2}\right)^2\sin^2\theta\right]}d\theta$$

$$=\frac{a(2-e^2)}{E(e,\frac{1}{2}\pi)}\int_0^{\frac{1}{2}\pi}\sqrt{\left[1-\left(\frac{e^2}{2-e^2}\right)^2\sin^2\theta\right]}d\theta\ldots.(1).$$

Representing by c the modulus of the elliptic integral in (1), we have $D=a(2-e^2)[E(c,\frac{1}{2}\pi)\div E(e,\frac{1}{2}\pi)]$, which is the average length required.

III. Solution by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Let $2r$=any diameter.

Then $2r=2\sqrt{x^2+y^2}=2\sqrt{b^2+e^2x^2}$ since $y^2=\frac{b^2}{a^2}(a^2-x^2)$ and $\frac{a^2-b^2}{a^2}=e^2$.

$$\therefore\ \triangle=\text{average length}=2\int_0^a\sqrt{b^2+e^2x^2}dx\div\int_0^a dx$$

$$\triangle=\frac{2}{x}\int_0^a\sqrt{b^2+e^2x^2}dx=\frac{2}{a}\left[\frac{x}{2}\sqrt{b^2+e^2x^2}+\frac{b^2}{2e}\log\left\{x+\frac{\sqrt{b^2+e^2x^2}}{e}\right\}\right]_0^a$$

$$\therefore\ \triangle=a+\frac{b^2}{ae}\log\left\{\frac{a(1+e)}{b}\right\}.$$

NOTE—Professor Matz gave a similar solution obtaining the same result in a different form. He has now furnished six different solutions to this problem. Three of these solutions give, for $e=\frac{1}{2}$, the average length of a diameter $=\frac{263}{250}b$, two give the average$=\frac{228}{250}a$, the result in the above solution, and one gives average$=\frac{268}{250}b$. These results are due to different interpretations of the problem. It occurs to us that the correct solution is obtained by considering the number of diameters proportional to the circumference of the ellipse. Taking

any diameter and having it pass through all possible values within proper limits by varying the ordinate or abscissa will give the sum of all the diameters. Dividing the sum of all the diameters by the number, which is equal to the circumference of the ellipse, will give the average diameter. The difference in the results $\frac{263}{250}b$ and $\frac{228}{250}a$ is about $\frac{1}{1000}b$. ED.]

7. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College. New Windsor, Maryland.

A letter is known to have come either from *Oshkosh* or *Ashland*. The only two consecutive letters legible on the postmark are *SH*. What is the probability that the letter received came from Oshkosh?

I. Solution by the PROPOSER.

Of the six pairs of consecutive letters in the word Oshkosh, *SH* are 2 pairs. Hence if the letter came from Oshkosh, the probability that *SH* was the legible pair is $\frac{2}{6}$, or $\frac{1}{3}$. If the letter came from Ashland, this probability is $\frac{1}{6}$. The *a posteriori* probability that the letter received came from Oshkosh, is, therefore, $P_O=\frac{\frac{2}{6}}{\frac{2}{6}+\frac{1}{6}}=\frac{2}{3}$; and that it came from Ashland, is $P_A=\frac{\frac{1}{6}}{\frac{2}{6}+\frac{1}{6}}=\frac{1}{3}$.

$\therefore P_O+P_A=\frac{2}{3}+\frac{1}{3}=1$.

Note.—In this connection, the following problem is appropriate and interesting: A letter is known to have come either from *Sing Sing* or *Lansing*. The only four consecutive letters legible on the postmark are *SING*. What is the probability that the letter received came from Sing Sing?

Answer: $P_S+P_L=\frac{8}{13}+\frac{5}{13}=1$.

II. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

According to the arrangement of the postmarks, *sh* of Ashland and the first *sh* of Oshkosh would be found in the left portion of the postmark, and the last *sh* of Oshkosh would be found in the right portion or near the top of the postmark.

There will, therefore, be two cases:—

(1) When a letter or figure of the date indicates the position of postmark.

(2) When the position of postmark cannot be determined.

Case 1. If *sh* is found in the right portion of postmark, the chances in favor of *Oshkosh* are $\frac{1}{0}$, or *infinity*; *i. e.* the letter came from Oshkosh.

If *sh* is found in the left portion, since the names of both places have the same number of letters and *sh* in both names is preceded only by the initial letter, the chances are equally divided, or $\frac{1}{1}$.

Case 2. In this case, since there are two *sh*'s in Oshkosh and only one in Ashland, the chances in favor of Oshkosh are 2 to 1.

III. Solution by G. B. M. ZERR, Principal of High School, Staunton, Virginia.

Since *sh* is found twice in Oshkosh and once in Ashland, the probability that the word is Oshkosh is $\frac{2}{3}$.